# PERMUTATION CODING AND MFSK MODULATION FOR FREQUENCY SELECTIVE CHANNEL

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**Abstract** – In frequency selective channel, some frequencies may be in deep fade. Inherent frequency diversity in multilevel frequency shift keying is exploited by a coding technique called permutation code, which ensures that the frequencies occur equally often. Information bits are uniformly spread over the frequency band. The performance of permutation codes and 4FSK in terms of bit error rate is compared with convolutional codes over 50 ns and 100 ns delay spread channel.

**Keywords** – Permutation arrays, Multilevel frequency shift keying, frequency selective channel.

## I Introduction

Multilevel frequency shift keying (MFSK) is an attractive modulation scheme for wireless radio system. The advantages include constant envelope and inherent frequency diversity. Frequency-hopping spread spectrum system (FHSS) with binary frequency shift keying (BFSK) is employed in the Bluetooth system [2]. FHSS/BFSK and FHSS/4FSK are supported in IEEE 802.11 wireless LAN standard [6].

Coding and decoding techniques for MFSK modulation have been widely studied. In [5] for instance, the authors examine and compare the performance of convolutional, Turbo and Reed-Solomon codes over MFSK in a frequency selective channel. Permutation trellis codes are proposed by Ferreira and Han Vinck in [4] to combat partial band interference and impulse noise. The main goal of this paper is to show that permutation *block* code is also a good candidate. Simulation results show that this simple and effective coded-modulation technique performs better than a much more complex convolutional-coded scheme in some signal to noise region.

The channel selectively attenuates one or more frequencies and sometimes causes erasures at demodulator output. Permutation codes are designed to recover the erasures by imposing the condition that each symbol appear exactly once in each codeword. The combination of permutation code and MFSK provides efficient frequency diversity..

Construction and examples of permutation codes can be found in [1, 3, 4].

In the next section, we describe the system model and a non-coherent receiver for a coded-MFSK modulation scheme. Examples of permutation codes are provided in Section III. Section IV contains the channel model and the parameters used in simulation. The simulation results are discussed in Section V. The last section is the conclusion.

## II System Description

In MFSK, we modulate the $k$th symbol, $k = 1, 2, \ldots, K$, by a sinusoidal signal,

$$s_k(t) = \sqrt{2E_s/T_s}\cos(2\pi f_k t), \quad 0 \le t \le T_s,$$

where $E_s$ is the symbol energy, $T_s$ the symbol duration, and $f_k = f_0 + (k-1)/T_s$ the frequency. The frequency separation is $1/T_s$. The $K$ sinusoidal signals are mutually orthogonal. We will use integers $1, 2, \ldots, K$ to denote the $K$ frequencies.

A coded-MFSK system can be represented by an $M \times N$ matrix $C = [c(i,j)]$ whose rows represent the codewords, where $M$ is the total number of messages and $N$ is the block length. The entries in the code matrix $C$ are integers $1, 2, \ldots, K$. We send out the $i$th message $(i = 1, \ldots, M)$ by transmitting the signal

$$m_i(t) = \sum_{j=0}^{N-1} s_{c(i,j)}(t - jT_s), \quad 0 \le t \le NT_s.$$

We assume that the channel is slowly varying and frequency selective. The received signal is

$$r(t) = \sum_{j=0}^{N-1} a_{c(i,j)}\sqrt{2E_s/T_s}\cos(2\pi f_{c(i,j)}(t - jT_s) + \theta_j) + n(t).$$

where $a_{c(i,j)}$ is the attenuation associated with frequency $c(i,j)$, $\theta_j$ is a uniform random variable distributed between 0 and $2\pi$, and $n(t)$ is additive white Gaussian noise.

The received signal is demodulated non-coherently by an envelope detector [8]. Let $W(j,k)$ be the energy detected during the $j$th symbol duration and at the $k$th frequency, for $j = 1,\ldots,N$ and $k = 1,\ldots,K$,

$$W(j,k) = \left(\int_{(j-1)T_s}^{jT_s} r(t)\sqrt{\frac{2}{T_s}}\cos(2\pi f_k t)\,dt\right)^2 + \left(\int_{(j-1)T_s}^{jT_s} r(t)\sqrt{\frac{2}{T_s}}\sin(2\pi f_k t)\,dt\right)^2.$$

Soft decoding is done by choosing the message $\hat{i}$ that maximizes the total energy,

$$\hat{i} = \arg\max_i \sum_{j=1}^{N} W(i, c(i,j)).$$

## III Permutation Codes

A permutation code $\mathcal{C}$ is a block code of length $K$ over the alphabet $\{1,\ldots,K\}$, with each symbol appears once and only once in a codeword. Thus every codeword is a permutation of $1,\ldots,K$. The number of codewords, $|\mathcal{C}|$, is a power of 2 less than $K!$. The code rate is

$$\frac{\log_2(|\mathcal{C}|)}{K\log_2(K)}.$$

It is easy to see that that the minimum Hamming distance of a permutation code is at least 2, and hence every permutation code can correct one erasure. If one of the frequencies in MFSK modulation suffers from deep fade, the erased symbol can be recovered after decoding.

We will list three examples of permutation codes. The alphabet are $\{1,2,3,4\}$ in all three examples, as they go along with 4FSK and are of most practical interests.

**Example 1:** $K = 4$, rate 1/2. The codewords are the rows in the matrix

$$C = \begin{bmatrix} 1&2&3&4\\ 1&2&4&3\\ 1&3&2&4\\ 1&3&4&2\\ 1&4&2&3\\ 1&4&3&2\\ 2&1&3&4\\ 2&1&4&3\\ 3&2&1&4\\ 3&2&4&1\\ 2&3&1&4\\ 2&3&4&1\\ 3&4&2&1\\ 3&4&1&2\\ 3&1&2&4\\ 3&1&4&2 \end{bmatrix}.$$

The minimum distance of this permutation code is equal to 2.

**Example 2:** $K = 4$, rate 3/8.

$$C = \begin{bmatrix} 1&2&3&4\\ 1&3&4&2\\ 1&4&2&3\\ 3&2&4&1\\ 4&1&3&2\\ 2&3&1&4\\ 2&4&3&1\\ 2&1&4&3 \end{bmatrix}$$

**Example 3:** $K = 4$, rate 1/4.

$$C = \begin{bmatrix} 1&2&3&4\\ 4&1&2&3\\ 3&4&1&2\\ 2&3&4&1 \end{bmatrix}$$

The minimum distance of this code is 4. This code can correct three erasures.

Example 1 and 2 are from [4]. Example 3 is obtained by simply cyclically rotating the codewords. They can be viewed as distance conservative and distance increasing mapping. See [4] for more details.

## IV Simulation Model

The performance is obtained by simulation of a coded 4FSK frequency-hopping system. The symbol duration is 1 $\mu$s. We use 4FSK modulation with 1 MHz frequency separation. Packets consisting of 200 random message bits are sent through the channel. We assume that the

Table 1: Tapped-delay-line parameters for indoor office test environments

| Tab no. | Channel A relative delay | Channel A average power | Channel B relative delay | Channel B average power |
|---|---|---|---|---|
| 1 | 0 ns | 0 dB | 0 ns | 0 dB |
| 2 | 50 ns | −3.0 dB | 100 ns | −3.6 dB |
| 3 | 110 ns | −10.0 dB | 200 ns | −7.2 dB |
| 4 | 170 ns | −18.0 dB | 300 ns | −10.8 dB |
| 5 | 290 ns | −26.0 dB | 500 ns | −18.0 dB |
| 6 | 310 ns | −32.0 dB | 700 ns | −25.2 dB |

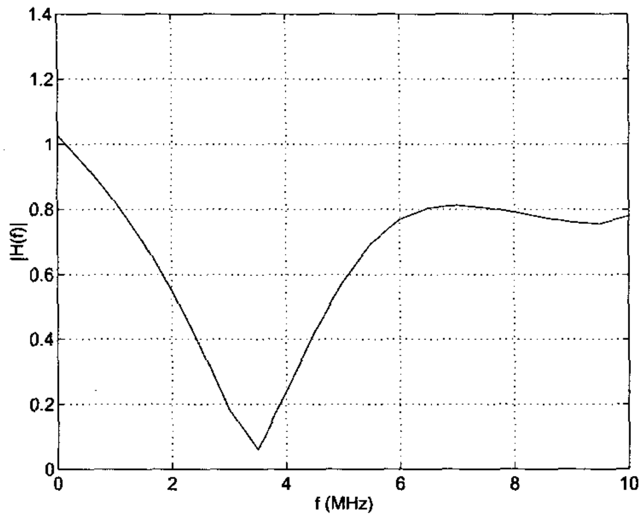


Figure 1: Channel response of 50ns channel

time variation of the channel is so slow that the channel response is constant during a packet. We hop to another carrier frequency after every packet. Inter-symbol interference is neglected.

We use tap-delay-line model to simulate an indoor frequency selective channel with 50ns and 100ns delay spread [7, p.620]. The relative delay and power are tabulated in Table 1. Typical channel responses of both channels are shown in Fig 1 and 2.

## V Simulation Results

We compare the performance of permutation code in Example 1 with rate-1/2 convolutional code with constraint length 4 and 7. The convolutional codes are decoded by soft-decision Viterbi algorithm.

The bit error rates are plotted against $E_b/N_0$, where $E_b$ is the energy per bit and $N_0$ is the one-sided power spectral density of the additive white Gaussian noise. The

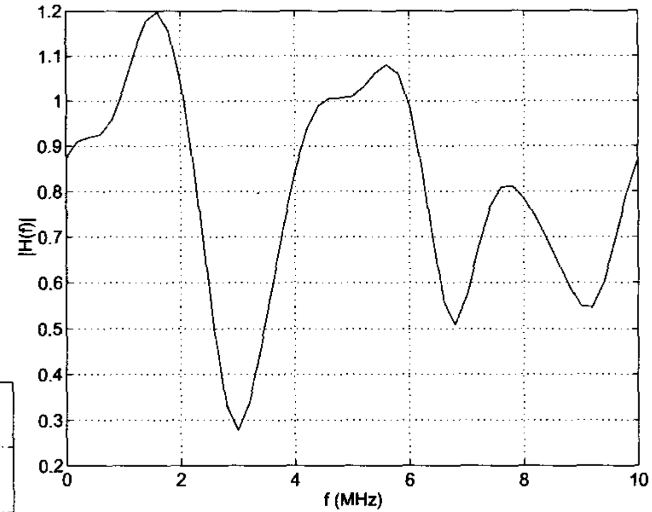


Figure 2: Channel response of 100ns channel

graphes for 50ns and 100ns channel are shown in Figure 3 and 4 respectively. Over the 50ns channel, the bit error rate of permutation code and a constraint length 3 convolutional code are very close for $E_b/N_0$ less than 20 dB. For higher $E_b/N_0$, the permutation code is better.

It is interesting to see that in the 100ns channel, the performance of permutation code is better than a constraint length 4 convolutional code. The bit error rate of a constraint length 7 convolutional code is also shown in the figure. The 64-state convolutional code exhibits an error floor around $5 \times 10^{-5}$, because the frequency selective channel causes error patterns that are uncorrectable by the convolutional code. The permutation code has coding gain of 4dB at bit error rate $10^{-5}$ over the 64-state convolutional code.

## VI Conclusion

We perform simulation experiment to compare the performance of permutation coding and other coded-modulation technique under frequency selective multipath channel. Although the code length of the permutation we simulate is only 4, the performance is even better than the constraint length 7 convolutional code when we are operating at sufficiently low bit error rate. Combination of permutation code and MFSK effectively provides frequency diversity by evenly spreading the bit energy to the MFSK symbols

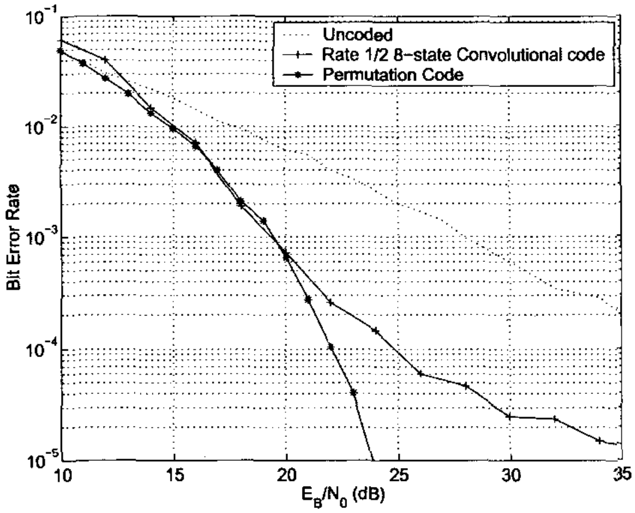


Figure 3: 50ns channel

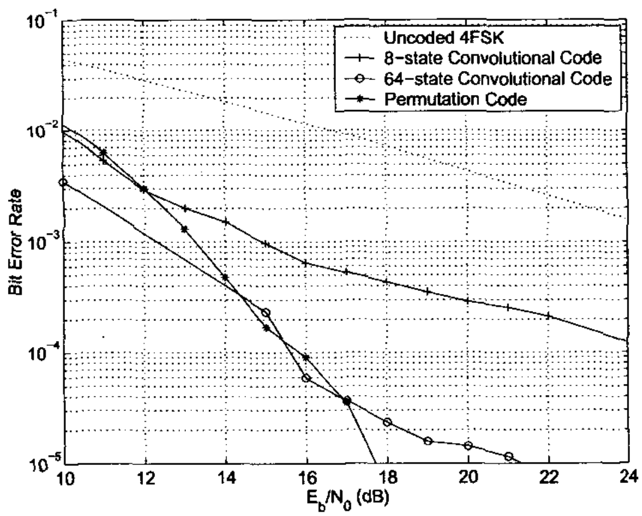


Figure 4: 100ns channel

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