High-Rate Permutation Coding with Unequal Error Protection

Kehinde Ogunyanda, Theo G. Swart, and Opeyemi O. Ogunyanda

Abstract—**Channel coding provides numerous advantages to digital communications. One of such advantages is error correcting capabilities. This, however, comes at the expense of coding rate, which is a function of the codebook's cardinality $|C|$ or number of coded information bits and the codeword length M. In order to achieve high coding rate, we hereby report a channel coding approach that is capable of error correction under power line communications (PLC) channel conditions, with permutation coding as the coding scheme of choice. The approach adopts the technique of unequal error correction for binary codes, but with the exception that non-binary permutation codes are employed here. As such, certain parts of the information bits are coded with permutation symbols, while transmitting other parts uncoded. Comparisons with other conventional permutation codes are presented, with the proposed scheme exhibiting a relatively competitive performance in terms of symbol error rate.**

Keywords—**Channel coding; High-rate codes; Permutation codes; Power line communications; Unequal error protection**

I. INTRODUCTION

POWER line communications (PLC), a technology that makes use of the ubiquitous electrical power line network to transmit digital data [1], is now widely considered as an alternative for wired communications. Although this helps in conveniently rolling out digital communications projects at cheaper rates, PLC is plagued with the challenges emanating from noise impairments due to the stochastic variation in the network load impedance. As such, various noise types, which are dependent on frequency, time and line types, have been associated with PLC channels. These noise types are categorised into three, namely background noise, narrowband noise and impulsive noise [2], [3]. Background noise is associated with various noise sources of low power whose power spectral density (PSD) decrease with rise in frequency [1]. Narrowband noise, otherwise known as frequency disturbance, results from radios of long and short waves that are in the same range as the PLC system. Impulsive noise, which affects multiple frequency components in the useful data, has a broadband PSD, and can yield multiple large envelopes [3], [4].

Frequency selective fading is another phenomenon that is typical of PLC, as in the case of wireless communication. Signal propagation in PLC does not only take a line of sight path from the transmitter to the receiver but also experiences additional echoes due to reflections generated by junctions in the cable layout, which are of different characteristic impedances [5].

Since 1830, when PLC technology started [6] by developing narrowband applications, a number of research works have featured in an attempt to make it a better form of communication [2], [7]–[9]. One of the significant works in PLC is the introduction of channel coding to combat the various noise types inherent in the channel. In [2], Vinck and Häring proposed the use of permutation coding, as a form of channel coding, in combating some of the PLC noise types. Afterwards, permutation coding has been widely studied for PLC, and it has been shown to be effective in tackling noise such as impulsive noise. In combating the effects of impulsive noise and narrowband interference, the authors in [10] proposed an encoded multi-tone frequency shift keying (MFSK) called permutation trellis coding. The coding entails translating a known convolutional code (CC) onto a permutation code. As such, the convolutional code was regarded as the base code, and its translated form maintains its Hamming distance attribute, if not improved. One disadvantage of permutation is the resulting low coding rate after using it to encode the information bits/symbols (i.e., user data).

A number of ways of achieving high coding rates already exist in the literature, some of which are puncturing, injection, and shortening [11], [12]. An attractive way of obtaining high rate is to retain some uncoded bits in the transmitted signals. This practice was presented in [13, Chapter 5], where only 8 bits, out of the codeword length of k-constrained codes, are encoded before transmission. The IBM codes with rates 16/17, 32/33, 48/49, 56/57, 72/73 and 80/81 have also been reported to use a similar approach [14]–[16]. Linear unequal error protection codes were reported in [17]–[19], where, in each codeword, certain digits are protected against a greater number of errors than the others. The approaches are, however, reported for coding with binary symbols. Non-binary version of these approaches were reported in [20] where Reed Solomon and non-binary Hamming codes are concatenated. These approaches consider the uncoded information bits as part of the codeword, and they are, therefore, referred to as the systematic part of the codeword.

In this work, we also employ the idea of encoding some parts of the transmitted information, but with the use of permutation codes, which are composed of non-binary symbols, while transmitting the remaining parts uncoded. As regards the already reported codes with unequal error correction, some parts of the codewords have more error protection than others,

Authors are with Center for Telecommunications, Department of Electrical and Electronic Engineering Science, University of Johannesburg, Johannesburg, South Africa (e-mail: ogunyanda@gmail.com, tgswart@uj.ac.za, opeyemiogunyanda@gmail.com).

this introduces complexity in generating such codebooks. This is another thing that makes the proposed scheme unique and different in that the uncoded information is not regarded as part of the codeword.

Our contribution here is to propose a simple way of achieving high rates in permutation coding by transmitting selected uncoded symbols with the coded ones, and to observe the extent to which this is practicable under PLC channel conditions. As far as we are aware, this is the first time such an approach is applied to a non-binary permutation coding scheme. Apart from PLC, unequal error protection schemes can also find use in other applications, such as satellite and terrestrial broadcasting of television signals with high-definition.

This paper is organised as follows. Section II provides a brief description of permutation coding and error correcting capability in relation to Hamming distance. The proposed high-rate permutation coding scheme is described in Section III, and, using a proposition, analyses on various coding rates obtainable from the proposed scheme are presented therein. Section IV is dedicated to results and discussion, while Section V concludes the paper.

II. PERMUTATION CODING

In digital communication, channel coding is achieved by introducing redundant/overhead information alongside the useful user data in order to mitigate against adverse channel conditions [21]. By definition, permutation coding is the representation of codewords, each containing M non-repetitive symbols selected from the set $\{0, 1, \ldots, M - 1\}$ [22]. In channel coding, permutation coding has been identified as being effective in combating most of the notorious noise types experienced in PLC [10], [23], [24].

The total number of possible codewords in a codebook $\mathcal{C}$ is called the cardinality $|\mathcal{C}|$, which is upper bound by [25]

$$|\mathcal{C}| \le \frac{M!}{(d_{\min} - 1)!}, \tag{1}$$

where $d_{\min}$ is the minimum Hamming distance. Given that permutation codewords $C_i = c_{i,k}$, where $i = 1, 2, \ldots |\mathcal{C}|$ and $k = 1, 2, \ldots M$, the term Hamming distance d is the number of locations where symbols $c_{i,k}$ and $c_{j,k}$ differ. The minimum value of d obtainable in $\mathcal{C}$ is regarded as its $d_{\min}$.

A codebook is said to be capable of correcting k errors, if its $d_{\min} \ge 2k+1$. In other words, a codebook's error correcting capability is upper bound by [26]

$$k \le \frac{d_{\min} - 1}{2}, \tag{2}$$

and its error-detecting capability is upper bound by

$$t \le d_{\min} - 1. \tag{3}$$

This implies that a codebook with large $d_{\min}$ is to be preferred if good error-correcting/detecting capability is needed. However, in order to achieve this, the codebook must be designed with large codeword length M. The implication of this is that error correction comes at the cost of adding

more redundancy in the code symbols. Also, the amount of redundancy directly relates to the coding rate. The lesser the overhead, the higher the rate of the code.

Table I presents some upper bound on the corresponding error-detecting-capability t in relation to cardinalities $|\mathcal{C}|$, codeword length M and minimum Hamming distance $d_{\min}$, based on (3).

III. HIGH-RATE PERMUTATION CODING

The rate of a codebook is defined as the ratio of uncoded bits to coded bits in the transmitted data stream [27]. With regards to non-binary block codes, such as permutations, the coding rate R is computed as a function of the cardinality $|\mathcal{C}|$, codeword length M and symbol size q. This is expressed as

$$R = \frac{\log_2 |\mathcal{C}|}{M \log_2 q}. \tag{4}$$

Example 1: Consider a permutation codebook

$$\{1230, 2301, 3012, 0123\}, \tag{5}$$

where $M = 4$, $d_{\min} = 4$ and $|\mathcal{C}| = 4$. The permuted symbols are selected from the set $U = \{0, 1, 2, 3\}$. This implies that the symbol size q of the codebook is 4. The code rate R of this codebook is computed, using (4) as

$$R = \frac{\log_2 |\mathcal{C}|}{M \log_2 q} = \frac{\log_2 4}{4 \log_2 4} = \frac{2}{8} = \frac{1}{4}. \tag{6}$$

The implication of this is that $\log_2 |\mathcal{C}| = 2$ information bits are mapped to M non-binary symbols whose bit number is $M \log_2 q = 8$. We can, therefore, infer from this example that 1 information bit is expanded into 4 bits, based on the codebook in (5). Although the codebook is capable of detecting $t \le 3$ bit errors, it is only achievable at the expense of a very low rate of $R = 1/4$.

Provided that a total of 10000 uncoded information bits are to be encoded and transmitted using the codebook in (5), we can infer that the user data is scaled up by a factor of 4, which is $1/R$. Hence, the overall length of the transmitted data stream will be 4×10000. However, provided that 2 out of every 2 bits are to be uncoded while the others are coded, this implies that the total length of the resulting data stream to be transmitted is $4 \times 5000 + 5000$. With this, the following proposition holds.

TABLE I
UPPER BOUNDS FOR k AND t OF PERMUTATION CODES

| $d_{\min} = 2$ | | $d_{\min} = 3$ | | $d_{\min} = 4$ | | $d_{\min} = 5$ | |
| $t \le 1$ | | $t \le 2$ | | $t \le 3$ | | $t \le 4$ | |
M	$|\mathcal{C}|$	M	$|\mathcal{C}|$	M	$|\mathcal{C}|$	M	$|\mathcal{C}|$
2	2						
3	6	3	3				
4	24	4	12	4	4		
5	120	5	60	5	20	5	5

Proposition 1: *Given that a permutation code with $|\mathcal{C}|$, M and q is used to encode a selected part of user data, while the remaining part is uncoded, the resulting rate is given by*

$$R' = R\frac{K}{K_c + RK_u}, \tag{7}$$

where R is the coding rate of the permutation code, K is the length of the user data before coding, K_c is the length of the coded user data, while K_u is the length of the uncoded part of the user data.

Proof: If K_c bits, out of K, are encoded with a code of rate R, this implies that K_c is scaled up by a factor of $1/R$. Since K_u is uncoded, the total length of the resulting data stream is $N = 1/R \times K_c + K_u$. Using the definition of rate, we can infer that

$$R' = \frac{K}{N} = \frac{K}{1/R \times K_c + K_u} \tag{8}$$

A. Algorithm for high-rate permutation coding

Henceforth, the proposed high-rate permutation coding/code shall be referred to as HRPC. A simple encoding algorithm is proposed for HRPC. A codebook with codeword length M, cardinality $|\mathcal{C}|$ and minimum Hamming distance $d_{\min}$ is first generated. Segment the user data into $n + m$ blocks, where $n = \log_2 |\mathcal{C}|$ (i.e. the number of bits that a conventional permutation code can map to M symbols), and m is the number of uncoded bits. To uniformly distribute burst of errors, interleave the uncoded segment with the coded segment in a deterministic manner and then transmit the resulting data stream.

At the receiving end, de-interleave and decode the encoded data using maximum likelihood approach. The user data is then assembled by concatenating the n decoded data with the received m uncoded data.

1) Codes with $M \neq 2^r$: Conventional modulators have constellations that are power of 2 (i.e. 2^r, where r is a positive integer). For instance, a differential quadrature amplitude shift keying modulator (DQPSK) has 2^2 constellations, and an 8DPSK modulator has 2^3. This implies that any permutation codebook, whose codeword length M is $\neq 2^r$, poses a challenge of unevenly distributed constellations after being modulated. For example, an $M = 5$ permutation code will, ideally, be modulated using an 8th order modulator (i.e., $2^3 = 8$ constellations). This implies that only 5, out of a total of 8 symbols, are useful, thereby causing the remaining 3 symbols to appear as *foreign symbols* in form of substitution errors. This phenomenon is described in detail in [8].

2) Modulation of HRPC symbols: We shall briefly describe 2 approaches to deal with the issue of *foreign symbols* in modulating $M \neq 2^2$ permutation codes. The first method entails the use of the approach reported in [28] by modifying the constellations of the modulator such that they match the permutation codeword length M. As such, we make use of an M constellations modulator instead of 2^r. We thus term this as modified modulator. We shall illustrate this using an example.

Example 2: Consider a permutation codebook

$$\{12340, 23401, 34012, 40123\}, \tag{9}$$

where $M = 5$. Since there are 5 symbols in the codebook, a $2^r = 8$ DPSK (i.e. 8DPSK, for instance) is needed to modulate such codebook, where $r = 3$. This, therefore, causes symbols 5, 6 and 7, which are not part of the codebook, to feature in the demodulated signal due to channel errors. In order to prevent this, we can modify the DPSK constellation points to be exactly the same as the codeword length of $M = 5$ (i.e., $M \neq 2^r$). This, thus, ensures that only symbols $0, 1, \ldots 4$ feature in the demodulated signal, thereby getting rid of the *foreign symbols* 5, 6 and 7. In DPSK, such scheme is termed the 5DPSK modulator.

To employ this in an HRPC modulated system, n user data bits can be coded with the permutation codebook and modulated using the modified modulator (i.e., 5DPSK as in this example), while the uncoded m bits are directly modulated using the conventional 2^r (i.e., 8DPSK) modulator.

The other approach to address the issue of *foreign symbols* in modulating $M \neq 2^r$ permutation codes is to disperse all the code symbols such that the entire 2^r constellations of the conventional modulator are evenly utilised. This is possible by the use of a scheme called permutation coding with injections, as reported in [12]. We shall also illustrate this using an example.

Example 3: Consider a permutation codebook in (9) with symbol size $q = 5$ and $d_{\min} = 4$. Instead of using this code book in an 8DPSK modulator, it is possible to derive another codebook of the same length $M = 5$ by increasing the symbol size to $q = 8$ and generating a $d_{\min} = 8$ codebook, then truncating 3 columns from the codebook. This is as illustrated in Fig. 1.

This, therefore, yields an injection codebook of $M = 5$ and $d_{\min} = 5$. This can then be used to encode the n user bits in an HRPC system and modulate it alongside the uncoded m bits using the conventional 2^r constellations modulator (i.e., 8DPSK in this example). With this, there is no need to modify the modulator, and *foreign symbols* do not feature in the demodulated data since all symbols are represented in the codebook.

B. HRPC rate analysis

Based on the definitions of K, K_c and K_u in **proposition 1**, we know that $K = K_c + K_u$. We can then substitute K_c with $K - K_u$ in (7) as

Fig. 1. Coding with injections
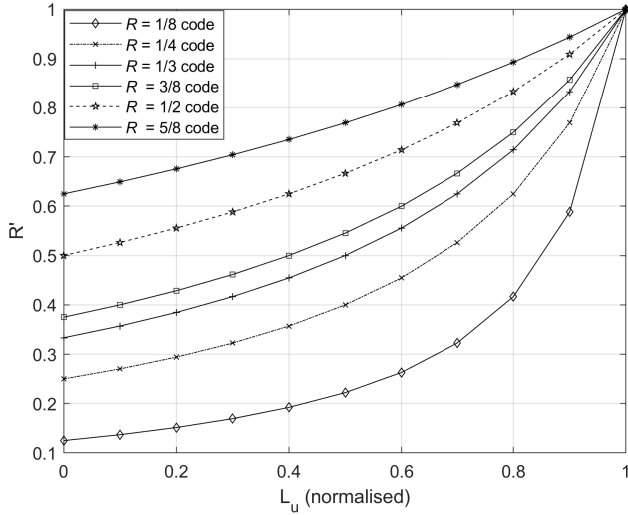
Fig. 2. Effective rates of codes with different R for various K_us

Fig. 3. Symbol error rate curves for DQPSK schemes under AWGN

$$R' = R\frac{K}{K - K_u + RK_u} = R\frac{K}{K - K_u(1 - R)}. \quad (10)$$

From (10), if $K_u = 0$, all user data is encoded by the permutation code and no uncoded data is transmitted. Also, if $K_u = K$, none of the user data is coded, and, as such, the rate is 1. With this, we can regard K_u as a fraction of K which can range from 0 to K. Fig. 2 shows the possible values of R' for various codes with different R, using a normalised K.

As K_u increases, R' also increases and vice versa. Also, as the normalised K_u approaches 1, the system's behaviour tends towards that of an uncoded system.

IV. EVALUATION AND PERFORMANCE RESULTS

The channel of evaluation in this work are focused on frequency selective fading, impulsive noise and background noise. These are typical of a PLC channel. Due to the time-varying frequency-selective nature of the PLC channel, it can be modeled as a fading channel having such characteristics with various noise sources. As such, a Rayleigh fading channel is adapted by tuning its properties to suit the PLC channel characteristics. The model used here is composed of six fading paths, each constituting a group of multipath components received at about the same delay. Details about such a channel model can be accessed from [27], [29]–[31], [31], [32]. The impulsive noise is modeled as impulses of equal height, having a fixed noise density whose value is between 0 and 1. The background noise is modeled as additive white Gaussian noise (AWGN) with deviation 1 and mean 0. To ensure fair comparisons, rate compensation is enforced on all the simulations considered.

The first set of evaluations done involves the use a differential phase shift keying (DPSK) modulator of order 4 (i.e. 4DPSK or DQPSK), in an orthogonal frequency division multiplexing (OFDM) system, to modulate the user data at different values of K_u. The coding aspect makes use of an
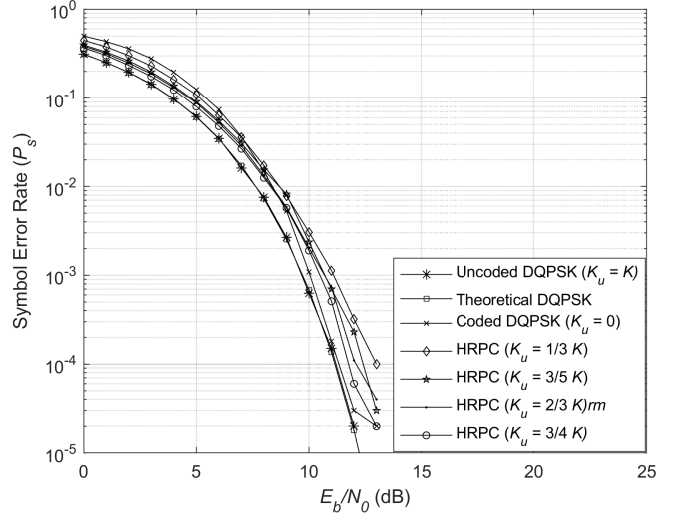
$M = 4$, $d_{\min} = 3$ permutation code. This codebook has a code rate of $R = 0.5$, as computed from (4).

With 1000000 user data bits, we consider for a case of $K_u = K$ (i.e., 1000000 bits), where the entire user data is uncoded. This scenario can be regarded as an uncoded DQPSK case. Other scenarios of HRPC with $K_u = 1/3\ K$ (i.e., 330000 bits), $K_u = 3/5\ K$ (i.e., 600000 bits), $K_u = 2/3\ K$ (i.e., 667000 bits), $K_u = 3/4\ K$ (i.e., 750000 bits) and $K_u = 0$ were considered as well. When $K_u = 0$, the situation is considered as when all user data is encoded with the $M = 4$, $d_{\min} = 3$ permutation code. Uncoded DQPSK, with no noise, was simulated alongside the schemes as a means of validating the correctness of the simulation. These variables were substituted in (7) to get the corresponding rates R' which are shown in Table II.

Figure 3 shows the simulations of the above schemes under ordinary AWGN channel conditions. It is noticeable from the figure that rate compensation enables the schemes to have relatively close performances. A fully coded scheme should, ideally have better performance than the HRPC schemes, its lower rate also partly impairs its performance. Likewise, HRPC schemes should exhibit poorer performances, but their higher rates also positively contribute to their performances. Based on this, we can affirm that better performance is traded for a higher rate and vice versa. The situation is, however, not the same if other forms of noise, other than AWGN, are considered, since different codes have different characteristics with respect to the channel type considered. As such, the rest of the simulation results shall showcase the significance of coding schemes with high rate under other PLC channel conditions.

Figure 4 shows various DPSK cases considered under AWGN and frequency selective fading channel conditions. Between $E_b/N_o = 0$ and $E_b/N_o = 10$ dB, the case where $K_u = K$ (i.e., uncoded DQPSK) is seen to be slightly better than the rest of the schemes. However, the $K_u = 0$ HRPC, followed by the $K_u = 1/3\ K$ HRPC, outperforms all the

TABLE II
EFFECTIVE RATES FOR $M = 4$ HRPC CODES

K_u	0	1/3 K	3/5 K	2/3 K	3/4 K	K
R'	0.5	0.6	0.714	0.75	0.8	1
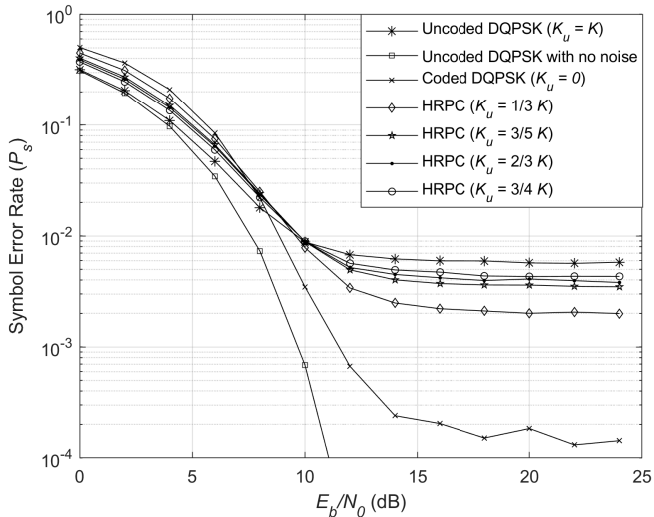
Fig. 4. Symbol error rate curves for DQPSK schemes under AWGN and frequency selective fading channel
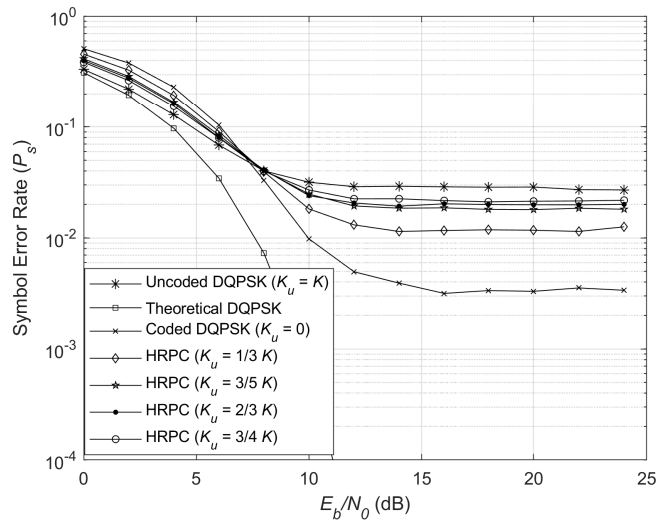

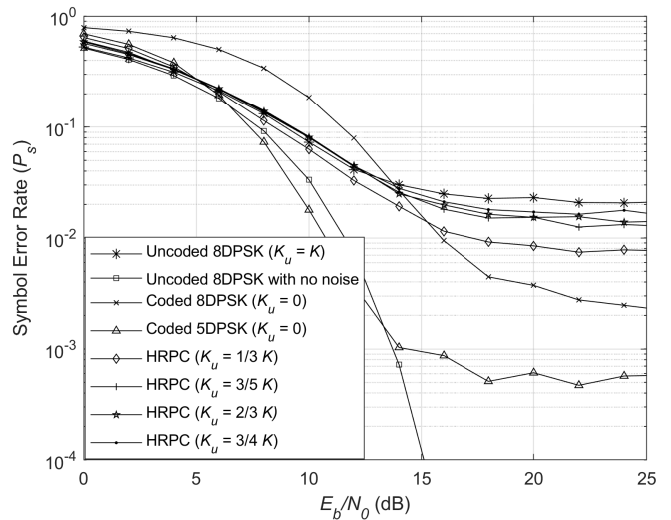Fig. 5. Symbol error rate curves for DQPSK schemes under AWGN, frequency selective fading and impulsive noise channel

Fig. 6. Symbol error rate curves for modified DPSK modulator schemes under AWGN and frequency selective fading channel

other schemes between $E_b/N_o = 10$ and $E_b/N_o = 25$, while $K_u = K$ HRPC is the worst of all. As K_u increases, the rate also increases, but at the expense of poorer performances.

In the case of AWGN, frequency selective fading and impulsive noise, the performance gap between the $K_u = 0$ scheme and the $K_u = 1/3$ K HRPC scheme has reduced in the range $E_b/N_o = 10$ dB and $E_b/N_o = 25$ dB, and all other HRPC schemes are seen to outperform the ordinary uncoded 8DPSK scheme, as shown in Fig. 5. Also, at $E_b/N_o < 10$ dB, the HRPC schemes outperform the fully coded scheme (i.e., $K_u = 0$).

We also considered different scenarios for 8DPSK where $K_u = K, 1/3\ K, 3/5\ K, 2/3\ K$ and 0 with the use of an $M = 5$ permutation code; the modified modulator described in Section III-A is employed in the proposed HRPC schemes. As such, we make use of a 5 constellations (i.e. 5 DPSK) modulator, instead of 8, for the coded part, while 8DPSK is used for the uncoded part. According to (4), $R = 0.4$. Table III shows the effective rates of all the HRPC schemes, as computed from (7).

Figure 6 presents the various cases considered in the modified modulator approach, under AWGN and frequency selective fading channel conditions. The fully coded scheme with the modified 5DPSK modulator is seen to outperform all other schemes, followed by the $K_u = 1/3$ K HRPC. The fully coded scheme with 8DPSK modulator is only able to outperform the $K_u = 1/3$ K HRPC at $E_b/N_o > 15$ dB.

Under AWGN, frequency selective fading and impulsive noise, all the proposed HRPC schemes are seen to outperform the ordinary uncoded scheme between $E_b/N_o = 10$ dB and

$E_b/N_o = 25$ dB, as evident in Fig. 7. The fully coded scheme (i.e., $K_u = 0$) with 8DPSK modulator is the worst performing scheme between $E_b/N_o = 0$ dB and $E_b/N_o = 17$ dB, but the best at $E_b/N_o > 17$.

For the injection coding method, we make use of an $M = 5$ and $d_{\min} = 4$ injection codebook in the proposed HRPC schemes. Figure 9 presents the various cases considered here, where AWGN and frequency selective fading are involved. Here, the performance gap between the proposed HRPC schemes and the fully coded one (i.e., $K_u = 0$), at $E_b/N_o > 14$, is relatively small as compared to the scenario where the modified modulator is used. All the coded schemes outperform the uncoded scheme in this region as well. When AWGN, frequency selective fading and impulsive noise are considered, the uncoded scheme (i.e., $K_u = K$) loses its best

TABLE III
EFFECTIVE RATES FOR $M = 8$ HRPC CODES

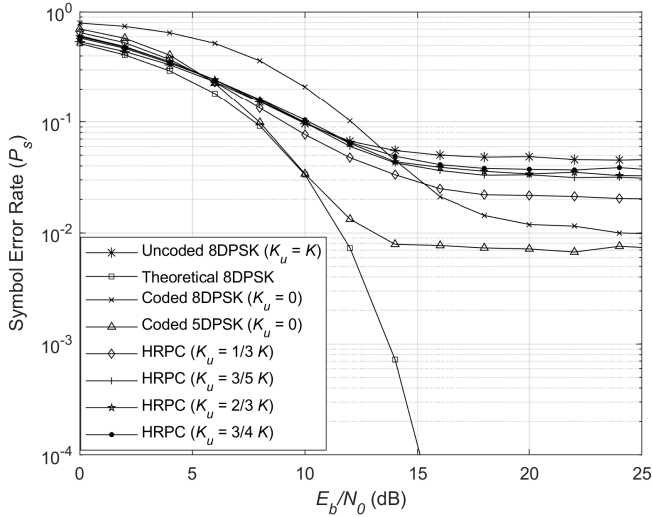| K_u	0	1/3 K	3/5 K	2/3 K	3/4 K	K
R'	0.4	0.5	0.625	0.667	0.727	1

Fig. 7. Symbol error rate curves for modified DPSK modulator schemes under AWGN, frequency selective fading and impulsive noise channel
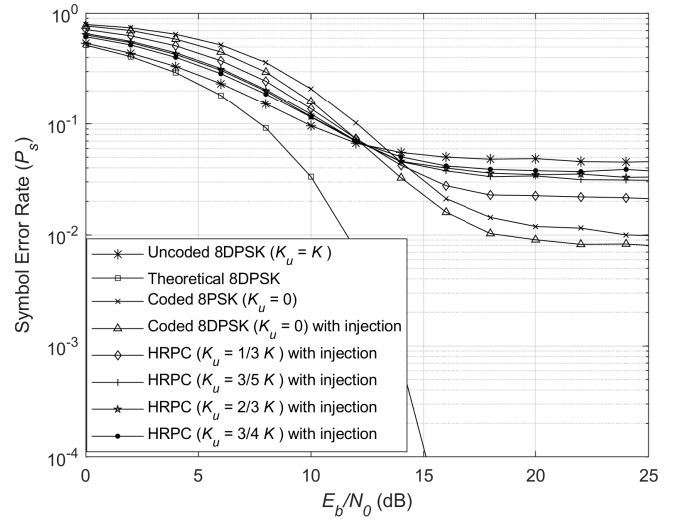

Fig. 8. Symbol error rate curves for injection coding schemes under AWGN, frequency selective fading and impulsive noise channel

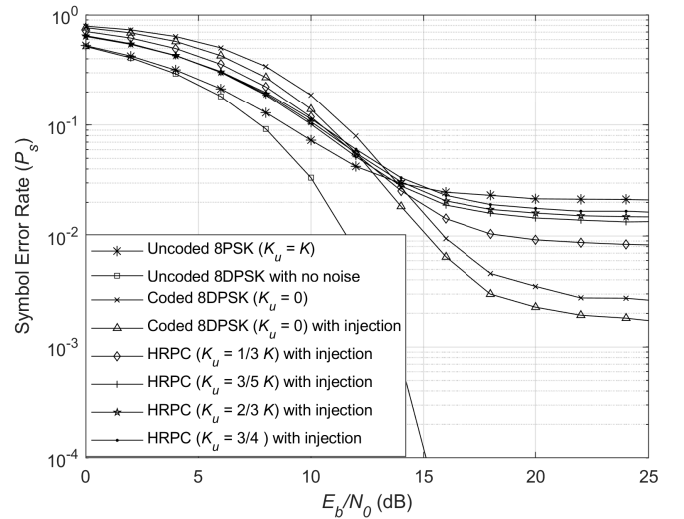Fig. 9. Symbol error rate curves for injection coding schemes under AWGN and frequency selective fading channel

performance at $E_b/N_o > 14$, as seen in Fig. 8. Also, the $K_u = 0$ (i.e., fully uncoded) scheme is unable to outperform the $K_u = 1/3$ K HRPC scheme at all values E_b/N_o.

As presented in **Proposition 1**, we observe that an increase in K_u yields an increase in R. However, a high-rate coding scheme offers good performance at very low E_b/N_o, but its performance is traded off at higher E_b/N_o values as evident in the simulation results from Figs. 4 to 8. Also, as K_u approaches K, the system's performance tends to become closely related to that of an uncoded scheme. Although a fully coded scheme (i.e., $K_u = 0$) outperforms the HRPC ones at higher E_b/N_o regions, the HRPC schemes are to be desired when high speed transmission is needed.

It should be noted that these results are shown for very severe channel conditions to better illustrate the performance difference between the codes, and with this, it shows that the limited communications will be possible for short periods.

V. CONCLUSION

In this paper, we have reported a simple way of achieving high-rate permutation coding schemes by partially encoding the user data with permutation codes of codeword length M and symbol size q. The significance of the partially coded schemes is evident at low E_b/N_o regions, where their performances are slightly better than the fully coded schemes. Also, at high E_b/N_o region, the partially coded schemes exhibit better performance than cases where the user data is fully uncoded. Smart meters, which are key components of smart grid, and home area networks are some of the applications that can make use of the proposed schemes.

REFERENCES

[1] J. Meng, "Noise analysis of power-line communications using spread-spectrum modulation," *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1470–1476, 2007. [Online]. Available: https://doi.org/10.1109/TPWRD.2007.900301

[2] A. J. H. Vinck and J. Häring, "Coding and modulation for power-line communications," in *Power Line Commun. and Its Applicat. (ISPLC), 2000 4th IEEE Int. Symp.*, Limerick, Ireland, Apr. 2000, pp. 265–271.

[3] V. N. Papilaya, T. Shongwe, A. J. H. Vinck, and H. C. Ferreira, "Selected subcarriers QPSK-OFDM transmission schemes to combat frequency disturbances," in *Power Line Commun. and Its Applicat. (ISPLC), 2012 IEEE Int. Symp.*, Beijing, China, Mar. 27–30, 2012, pp. 200–205. [Online]. Available: https://doi.org/10.1109/ISPLC.2012.6201327

[4] H. C. Ferreira, A. J. H. Vinck, T. G. Swart, and I. de Beer, "Permutation trellis codes," *IEEE Trans. Commun.*, vol. 53, pp. 1782–1789, Nov. 2005. [Online]. Available: https://doi.org/10.1109/TCOMM.2005.858683

[5] M. Zimmermann and K. Doster, "A multi-path signal propagation model for the power line channel in the high frequency range," in *Proc. IEEE Int. Symp. Power Line Commun.*, Lancaster, UK, 1999, pp. 45–51.

[6] M. Tlich, H. Chaouche, A. Zeddam, and P. Pagani, "Novel approach for PLC impulsive noise modelling," in *Power Line Commun. and Its Applicat. (ISPLC), 2009 IEEE Int. Symp.*, 2009, pp. 20–25. [Online]. Available: https://doi.org/10.1109/ISPLC.2009.4913397

[7] M. Zimmermann and K. Dostert, "A multipath model for the powerline channel," *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 553–559, 2002. [Online]. Available: https://doi.org/10.1109/26.996069

[8] K. Ogunyanda, A. D. Familua, T. G. Swart, H. C. Ferreira, and L. Cheng, "Evaluation of mixed permutation codes in PLC channels, using Hamming distance profile," *Telecommun. Syst.*, vol. 65, no. 1, pp. 1–11, 2016.

[9] T. G. Swart and H. C. Ferreira, "Analysis of permutation distance-preserving mappings using graphs," in *Int. Symp. on Commun. Theory and Applicat.*, 2007.

[10] H. C. Ferreira and A. J. H. Vinck, "Interference cancellation with permutation trellis codes," in *Vehicular Technol. Conf., 2000. IEEE-VTS Fall VTC 2000. 52nd*, vol. 5, 2000, pp. 2401–2407. [Online]. Available: https://doi.org/10.1109/VETECF.2000.883295

[11] J. I. Hall, "Notes on coding theory," Lecture notes. [Online]. Available: https://users.math.msu.edu/users/halljo/classes/codenotes/mod.pdf

[12] P. J. Dukes, "Coding with injections," *Designs, Codes and Cryptography*, vol. 65, no. 3, pp. 213–222, 2012.

[13] K. A. S. Immink, *Codes for Mass Data Storage Systems*. The Netherlands: Shannon Foundation Publishers, 1999.

[14] J. L. Sonntag, *Apparatus and Method for Increasing Density of Run-length Limited Codes without Increasing Error Propagation*. US Patent 5,604,497, Feb. 1997.

[15] P. McEwen, B. Zafar, and K. Fitzpatrick, *High Rate Runlength Limited Codes for 8-bit ECC Symbols*. US Patent 6,201,485, Mar. 2001.

[16] A. McClellan, *Runlength-limited Code and Method*. US Patent 6,285,302, Sept. 2001.

[17] B. Masnick and J. Wolf, "On linear unequal error protection codes," *IEEE Trans. Inf. Theory*, vol. IT-3, no. 4, p. 600–607, 1967. [Online]. Available: https://doi.org/10.1109/TIT.1967.1054054

[18] R. Morelos-Zaragoza, T. Kasami, S. Lin, and H. Imai, "On block-coded modulation using unequal error protection codes over rayleigh-fading channels," *IEEE Trans. Commun.*, vol. 46, no. 1, pp. 1–4, 1998. [Online]. Available: https://doi.org/10.1109/26.655394

[19] R. Morelos-Zaragoza and S. Lin, "Qpsk block-modulation codes for unequal error protection," *IEEE Trans. Inf. Theory*, vol. 41, no. 2, pp. 576–581, 1995. [Online]. Available: https://doi.org/10.1109/18.370154

[20] ——, "On a class of optimal nonbinary linear unequal-error-protection codes for two sets of messages," *IEEE Trans. Inf. Theory*, vol. 40, no. 1, pp. 196–200, 1994. [Online]. Available: https://doi.org/10.1109/18.272481

[21] S. Haykin, *Communication systems*, 4th ed. Wiley, 2001.

[22] O. O. Ogunyanda, T. Shongwe, and T. G. Swart, "Permutation coding with injections for modified pam system," *Int. J. Electron. Telecommun.*, vol. 6, no. 3, pp. 399–404, 2020.

[23] P. J. Dukes, F. Ihringer, and N. Lindzey, "On the algebraic combinatorics of injections and its applications to injection codes," *IEEE Trans. Inf. Theory*, vol. 66, no. 11, pp. 6898–6907, 2020. [Online]. Available: https://doi.org/10.1109/TIT.2020.3005544

[24] T. G. Swart and H. C. Ferreira, "Decoding distance-preserving permutation codes for power-line communications," in *IEEE AFRICON 2007*, Windhoek, Namibia, 2007, pp. 1–7. [Online]. Available: https://doi.org/10.1109/AFRCON.2007.4401563

[25] M. Deza and S. A. Vanstone, "Bounds on permutation arrays," *J. Stat. Planning and Inference*, vol. 2, no. 2, pp. 197–209, 1978. [Online]. Available: https://doi.org/10.1016/0378-3758(78)90008-3

[26] H. C. Ferreira, H. M. Grové, O. Hooijen, and A. J. H. Vinck, "Power line communication," *Wiley Encyclopedia of Electrical and Electronics Engineering*, 2001.

[27] A. Patapoutian, J. Stander, P. McEwen, B. Zafer, and J. Fitzpatrick, *Rate 32/33 (D=0, K-6) run length limited modulation code having optimized error propagation*. US Patent 6,184,806, Feb. 2001.

[28] K. Ogunyanda, A. D. Familua, T. G. Swart, H. C. Ferreira, and L. Cheng, "Permutation coding with differential quinary phase shift keying for power line communication," in *IEEE PES Innovative Smart Grid Technol. European Conference*, Istanbul, Turkey, Oct. 12–15, 2014, pp. 1–6. [Online]. Available: https://doi.org/10.1109/ISGTEurope.2014.7028891

[29] C. Oestges and B. Clerckx, *MIMO Wireless Communications: From Real-World Propagation to Space-Time Code Design*. Academic Press, 2007.

[30] L. M. Correira, *Mobile Broadband Multimedia Networks: Techniques, Models and Tools for 4G*. Academic Press, 2006.

[31] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Selected Areas of Commun.*, vol. 20, no. 6, p. 1211–1226, 2002.

[32] MathWorks, "comm.ricianchannel system object," Accessed May 28, 2021. [Online]. Available: https://www.mathworks.com/help/comm/ref/comm.ricianchannel-system-object.html