Abstract of the Paper: The rank modulation scheme has been proposed for efficient writing and storing data in non-volatile memory storage. Error-correction in the rank modulation scheme is done by considering permutation codes. In this paper we consider codes in the set of all permutations on n elements, S_n, using the Kendall τ-metric. The main goal of this paper is to derive new bounds on the size of such codes. For this purpose we also consider perfect codes, diameter perfect codes, and the size of optimal anticodes in the Kendall τ-metric, structures which have their own considerable interest. We prove that there are no perfect single-error-correcting codes in S_n, where $n > 4$ is a prime or $4 \leq n \leq 10$. We present lower bounds on the size of optimal anticodes with odd diameter. As a consequence we obtain a new upper bound on the size of codes in S_n with even minimum Kendall τ-distance. We present larger single-error-correcting codes than the known ones in S_5 and S_7.

Preliminary definitions:

1. Permutation Codes: permutation code of length n is a non-empty subset (C) of S_n, the set of all permutations of $[n] := \{1, 2, \ldots, n\}$.

2. Adjacent transposition: Given a permutation $\pi := [\pi(1), \pi(2), \ldots, \pi(i), \pi(i+1), \ldots, \pi(n)] \in S_n$, an adjacent transposition, $(i, i+1)$, for some $1 \leq i \leq n$, is an exchange of the two adjacent elements $\pi(i)$ and $\pi(i+1)$ in π. The result is the permutation $[\pi(1), \pi(2), \ldots, \pi(i+1), \pi(i), \ldots, \pi(n)]$.

3. Kendall τ-metric: For two permutations $\sigma, \pi \in S_n$, the Kendall τ-metric between σ and π, $d_K(\sigma, \pi)$, is defined as the minimum number of adjacent transpositions needed to transform σ into π.

4. $P(n, d)$: The size of the largest permutation code of length n with minimum Kendal τ-distance d is denoted by $P(n, d)$.

5. Perfect code: For a given space $\mathcal{V}$ with a distance measure $d(.,.)$, a subset $\mathcal{C}$ of $\mathcal{V}$ is a perfect code with radius R if for every element $x \in \mathcal{V}$ there exists exactly one codeword $c \in \mathcal{C}$ such that $d(x, c) \leq R$. A perfect code with radius R is also called a perfect R-error-correcting code.

6. Ball of radius R: For a given space $\mathcal{V}$ with a distance measure $d(.,.)$ and for a point $x \in \mathcal{V}$, the ball of radius R centered at x, $B(x, R)$, is defined by $B(x, R) := \{y \in \mathcal{V} \,|\, d(x, y) \leq R\}$.

7. Anticode with diameter D: Let $\mathcal{V}$ be a space with a distance measure $d(.,.)$. For any $\mathcal{A} \subset \mathcal{V}$, the diameter of $\mathcal{A}$, denoted by $D(\mathcal{A})$, is equal to $max\{d(x, y) \,|\, x, y \in \mathcal{V}\}$. We also say that $\mathcal{A}$ is an anticode with diameter $D(\mathcal{A})$.

8. D–diameter perfect code: Let $\mathcal{V}$ be a space with a distance measure $d(.,.)$. If there exists a code $C \subseteq \mathcal{V}$ with minimum distance $D+1$ and an anticode A with diameter D such that $|C| \cdot |A| = |\mathcal{V}|$, then C is called a D-diameter perfect code.